Report of working group 1.

▪ Existing curricula for calculus and linear algebra in the test institutions.

All the 10 test institutions provided the syllabi of the courses relevant to the subjects of the project: calculus I and linear algebra I. The study of these documents showed that there are many differences between the institutions:

- In some cases, calculus and linear algebra are not taught in separate courses but appear as modules inside a general Mathematics course.
- The number of hours taught each semester vary and in consequence also the quantity of material covered in one course.
- Depending of the program in which the courses are given, the balance between the theoretical and the computational subjects vary. Let us consider the subject of vector spaces in linear algebra as an illustration of this point: some courses concentrate on solving systems of linear equations and computing with matrices and do not cover vector spaces, some courses cover vectors spaces over the real numbers and some abstract vector spaces.

The following comparative tables summarize the subjects covered in calculus and in linear algebra.

	HAC tools	HAC calc.1	HAC calc.2	HIT	UoP math 1	UoP math 2	UoM Math 2	UC calc.1	UC calc. 2	FINKI	NUUz anal. 1	NUUz anal.2	TUIT math 1	TUIT math 2	KEEI math 1	KEEI math2	HMU
generalities on sets and numbers																	
basic set theory				v				v			v						v
axioms of reals upper and lower bounds		v			v						v						
numerical sequences																	
sequences			v	v	v			v			v	v	v		v		
limits of sequences			v	v	v			v			v	v	v		v		
generalities on functions																	
functions basic notions	v			v	v		v			v	v		v				v
elementary functions and their inverses (graph properties)		v		v						v	v						v
limits of functions																	
limits and continuity (computation)	v			v	v		v	v	v	v	v				v		
limits (theory)		v		v					v		v				v		
intermediate value theorem		v						v			v						
Weierstrass theorem								v			v						
uniform continuity											v						
derivation																	
derivative rules	v			v	v		v		v	v	v		v		v		v
derivative inverse functions		v		v											v		
rolle		v		v											v		
lagrange		v		v											v		
lhopital		v		v			v			v	v		v		v		
asymptotes		v					v		v		v				v		v
critical points and inflexion points		v			v		v		v	v	v				v		v
absolute extrema		v						v		v	v				v		v
differentials and applications		v		v			v				v		v		v		
monotonous functions		v			v			v	v	v	v				v		v
sketching of graph		v							v	v					v		v
inverse trigonometric functions		v													v		v
integrals																	
indefinite integral		v		v		v	v		v	v	v		v			v	v
methods of integration		v		v		v	v		v	v	v		v			v	v
definite integral		v		v		v	v		v	v	v		v			v	v
applications of definite integrals			v	v		v	v					v	v			v	
Darboux sumq Rieman sums											v	v					
improper integrals			v	v			v					v				v	
series																	
series			v		v			v				v	v				
taylor formula										v							
taylor series convergence			v				v						v				
power series			v									v	v				
differential equations																	
differential equations (first and second order)							v						v			v	
numerical analysis																	
Newton-Raphson			v														
fixed point iteration			v														

	HAC tools	HAC LA1	HAC LA2	HIT	UoP Math 1	UoM Math 1	UC	UC	FINKI	NUUZ LA	TUIT math 1	KEEI	HMU
analytic geometry													
vectors	v			v	v	v			v		v	v	v
scalar product	v			v	v	v			v		v	v	v
vector product	v			v	v	v			v		v	v	v
triple product					v						v		
lines and planes	v			v		v	v only lines		v		v	v	
number systems													
rings and field		v											
reals and complex	v				v	v	v only reals	v			v		v
systems of linear equations													
systems of linear equations		v		v	v	v	v 2-3 variables		v	v	v	v	
gauss elimination		v		v		v				v	v	v	
solution with matrices		v				v			v		v	v	v
Kronecker Capelli		v							v?	v		v	
cramer's rule		v				v				v			
matrices													
matrices		v		v	v	v	v		v	v	v	v	v
elementary matrices inverse matrices		v		v		v	v		v	v	v	v	v
transpose, symmetric matrices		v				v			v		v		v
rank		v							v	v	v		
LU factorization									v				
determinants		v		v	v	v	v		v	v	v	v	v
vector spaces													
vector spaces		v		v	v	v		v	v	v	v	v	
sum and intersections of vs		v		v		v			v	v	v		
linear combinations		v		v	v	v			v	v	v	v	
basis dimension		v		v	v	v			v	v	v	v	
linear transformation													
linear transformation			v	v				v	v	v	v		
change of coordinates-basis			v	v					v	v			
matrix representation of linear transformation			v	v					v	v			
eigenvalues eigenvectors			v	v					v	v	v		
characteristic polynomial			v	v					v	v	v		
applications			v	v					v				
SV decomposition									v				
jordan normal form													
Inner product spaces													
Inner product spaces			v										
Angles and orthogonality			v						v	v			
Gram Schmidt process			v						v	v			
orthogonal linear transformations			v						v				
Best approximation: least squares													
quadratic forms													
orthogonal matrices			v							v			
orthogonal diagonalization			v						v	v			
quadratic forms			v							v			
law of inertia			v							v			
hermitian, normal matrices			v							v			
singular value decomposition									v				

▪ The proposed curricula for the project

Because of all the differences mentioned above, it was clear that it would not be possible to write a curriculum that fits all the institutions. Instead we composed a curriculum for each course based on the subjects that were taught in most institutions and based on ~~the~~ a comparison with standard textbooks and courses in leading universities in Europe and the US. For each of the subjects in these curricula, teaching material will be developed in the project. The purpose of these curricula is not to replace the existing ones in the test institutions, but to prepare a bank of material to use according to the subjects taught. Test institutions which teach subjects that are outside of the scope of these curricula are free to develop additional teaching material to cover all of the subjects taught in their institutions.

To follow are the proposed curricula for both courses. The curricula were presented to the project's participants during the first management meeting in June 2019 and was accepted by all the test institutions' representatives. As a next step, based on these curricula, the subjects will be distributed among the test institutions to develop the required material for the pilots.

CALCULUS 1 SYLLABUS

1. Functions, generalities.
    - Basic notions on functions.
    - Graph of standard functions
2. Limit and continuity.
    - Intuitive definition
    - Computations of limits.
    - Sequences, convergence of sequences.
3. Derivation
    - Definition of derivative
    - Derivations rules.
    - Derivations of inverse functions for monotonous functions
    - Special functions
    - Application of derivative: linear approximation, rate of change, Newton Raphson method.
4. Behavior of function
    - Intermediate value theorem
    - Fermat theorem, critical points.
    - Absolute extrema. Optimization problems.
    - Rolle theorem and mean value theorem.
    - Increasing, decreasing functions. Local extrema.
    - Convexity. Torsion points.
    - Asymptotes
    - Sketch of graph.
5. The integral.
    - Indefinite integral (anti derivative)
    - Methods of integration

- Definite integral, definition. Introduction to Riemann sums.
- Fundamental theorem of calculus.
- Applications of the definite integral.
- Improper integrals.

LINEAR ALGEBRA SYLLABUS – A DRAFT

1. Vectors
    - Vectors
    - Dot product
    - Cross product (EE students)
    - Triple product (EE students)
    - Lines in the plane.
2. Systems of linear equations and matrices.
    - Systems of linear equations
    - Gauss elimination.
    - Matrices
    - Solution of systems of linear equations using matrices
    - Elementary matrices, inverse matrices
    - Transpose. Symmetric matrices
    - Rank
    - Determinants
    - Computation of determinant by cofactor expansion
    - Computation of determinant by row reduction.
3. Vector spaces
    - Definitions and examples.
    - Subspaces. Sum and intersections of subspaces
    - Linear combinations.
    - Basis, dimension.
    - Row space, column space and null space of a matrix. Rank theorem.
4. Linear transformations.
    - Definitions and examples.
    - Matrix representation of linear transformation
    - Change of coordinates/bases.
    - Eigenvalues and eigenvectors.
    - Characteristic polynomial
    - Diagonalization.
    - SVD